Formula Sheet

**Theorem 3.8**

If Y is a random variable with a geometric distribution,

**Definition 3.9**

A random variable Y is said to have a negative binomial probability distribution if and only if

**Theorem 3.9**

If Y is a random variable with a negative binomial distribution,

**Definition 3.10**

A random variable Y is said to have a hypergeometric   
probability distribution if and only if

where y is an integer 0, 1, 2,..., n, subject to the

restrictions and

**Theorem 3.10**

If Y is a random variable with a hypergeometric distribution,

**Definition 3.11**

A random variable Y is said to have a Poisson probability distribution if and only if

**Theorem 3.11**

If Y is a random variable possessing a Poisson distribution with parameter λ, then

**Definition 3.12**

The kth moment of a random variable Y taken about the origin is defined to be and is denoted by .

**Definition 3.13**

The kth moment of a random variable Y taken about its mean, or the kth central moment of Y , is defined to be and is denoted by

**Definition 3.14**

The moment-generating function for a random variable Y is defined to be  
. We say that a moment-generating function for Y exists if there exists a positive constant b such that is finite for

**Theorem 3.12**

If exists, then for any positive integer k,

In other words, if you find the kth derivative of   
with respect to t and then set, the result will be

**Definition 3.15**

Let Y be an integer-valued random variable for which where  
 The probability-generating function for Y is defined to be

for all values of t such that P(t) is finite,

**Definition 3.16**

The kth factorial moment for a random variable Y is defined to be

for all values of t such that P(t) is finite.

**Theorem 3.13**

If is the probability-generating function for an integer-valued random variable, Y , then the kth factorial moment of Y is given by

**Theorem 3.14**

**Tchebysheff’s Theorem** Let Y be a random variable with mean   
 and finite variance . Then, for any constant ,

**Definition 4.1**

The probability function for Y is given by

Which yields

**Theorem 4.1**

Properties of a Distribution Function1 If F(y)is a distribution function, then

1. is a nondecreasing function of y

[If and are any values such that , then .]

**Definition 4.2**

A random variable Y with distribution function F(y) is said to be continuous if F(y) is continuous, for

**Definition 4.3**

Let F(y) be the distribution function for a continuous random variable Y . Then f (y), given by

wherever the derivative exists, is called the probability density function for the random variable Y .

**Theorem 4.2**

Properties of a Density Function If f (y)is a density function   
for a continuous random variable, then

**Definition 4.4**

Let Y denote any random variable. If , the pth quantile of Y , denoted by , is the smallest value such that If Y is continuous, is the smallest value such that Some prefer to call , the 100pth percentile of Y

**Theorem 4.3**

If the random variable Y has density function , then the probability that Y falls in the interval is

**Definition 4.5**

The expected value of a continuous random variable Y is

provided that the integral exists

**Theorem 4.4**

Let be a function of Y ; then the expected value of is given by

provided that the integral exists.

**Theorem 4.5**

Let c be a constant and let be functions of a continuous random variable Y . Then the following results hold:

**Definition 4.6**

If , a random variable Y is said to have a continuous uniform probability distribution on the interval if and only if the density function of Y is

**Definition 4.7**

The constants that determine the specific form of a density function are called parameters of the density function.

**Theorem 4.6**

If and Y is a random variable uniformly distributed on the interval (, ), then

**Definition 4.8**

A random variable Y is said to have a normal probability distribution if and only if, for and the density function of Y is

,

**Theorem 4.7**

If Y is a normally distributed random variable with parameters , then

**Definition 4.9**

A random variable Y is said to have a gamma distribution with parameters and only if the density function of Y is

where

**Theorem 4.8**

If Y has a gamma distribution with parameters , then

**Definition 4.10**

Let ν be a positive integer. A random variable Y is said to have a chi-square distribution with ν degrees of freedom if and only if Y is a gamma-distributed random variable with parameters

**Theorem 4.9**

If Y is a chi-square random variable with ν degrees of freedom, then

**Definition 4.11**

A random variable Y is said to have an exponential distribution with parameter if and only if the density function of Y is

**Theorem 4.10**

If Y is an exponential random variable with parameter β, then

**Definition 4.12**

A random variable Y is said to have a beta probability distribution with parameters α > 0 and β > 0 if and only if the density function of Y is

where

**Theorem 4.11**

If Y is a beta-distributed random variable with parameters , then

**Definition 4.13**

If Y is a continuous random variable, then the kth moment about the origin is given by

The kth moment about the mean, or the kth central moment, is given by

**Definition 4.14**

If Y is a continuous random variable, then the moment-generating   
function of Y is given by

The moment-generating function is said to exist if there exists a constant such that is finite for

**Theorem 4.12**

Let Y be a random variable with density function and be a function of Then the moment-generating function for is

**Theorem 4.13**

**Tchebysheff’s Theorem Let Y be a random variable with finite mean μ and variance . Then, for any ,**

**Definition 5.1**

Let and be discrete random variables. The joint (or bivariate) probability

function for and is given by

**Theorem 5.1**

If and are discrete random variables with joint probability function then

where the sum is over all values that are assigned nonzero probabilities

**Definition 5.2**

For any random variables and , the joint (bivariate) distribution function is

**Definition 5.3**

Let and be continuous random variables with joint distribution function If there exists a nonnegative function such that

for all , then and are said to be jointly continuous random variables. The function is called the joint probability density function.

**Theorem 5.2**

If and are random variables with joint distribution function , then

**Theorem 5.2**

If and are jointly continuous random variables with a joint density

function given by, then

**Definition 5.4**

a Let and be jointly discrete random variables with probability function .Then the marginal probability functions of and , respectively, are given by

b Let and be jointly continuous random variables with joint density function .Then the marginal density functions of and , respectively, are given by

**Definition 5.5**

If and are jointly discrete random variables with joint probability function function . and marginal probability functions and , respectively, then the conditional discrete probability function of given is

Provided that

**Definition 5.6**

If and are jointly continuous random variables with joint density function , then the conditional distribution function of given is

**Definition 5.7**

Let and be jointly continuous random variables with joint density and marginal densities) and , respectively. For any such that , the conditional density of given is given by

and, for any such that , the conditional density ofgiven is given by